Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Precalculus or Calculus **In Exercises 1 and 2, determine whether the problem can be solved using precalculus or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning and use a graphical or numerical approach to estimate the solution.**

1. Find the distance between the points $(1, 1)$ and $(3, 9)$ along the curve $y = x^2$.

2. Find the distance between the points $(1, 1)$ and $(3, 9)$ along the line $y = 4x - 3$.

Estimating a Limit Numerically **In Exercises 3 and 4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.**

3. $\displaystyle\lim_{x\to 3} \frac{x-3}{x^2-7x+12}$

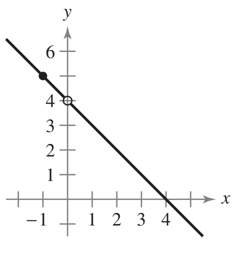
x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$				?			

4. $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

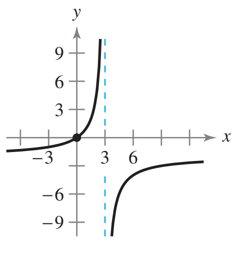
Finding a Limit Graphically **In Exercises 5 and 6, use the graph to find the limit (if it exists). If the limit does not exist, explain why.**

5. $h(x) = \dfrac{4x - x^2}{x}$



(a) $\displaystyle\lim_{x\to 0} h(x)$ (b) $\displaystyle\lim_{x\to -1} h(x)$

6. $g(x) = \dfrac{-2x}{x-3}$



(a) $\displaystyle\lim_{x\to 3} g(x)$ (b) $\displaystyle\lim_{x\to 0} g(x)$

Using the ϵ–δ Definition of a Limit **In Exercises 7–10, find the limit L. Then use the ϵ–δ definition to prove that the limit is L.**

7. $\displaystyle\lim_{x\to 1} (x+4)$

8. $\displaystyle\lim_{x\to 9} \sqrt{x}$

9. $\displaystyle\lim_{x\to 2} (1-x^2)$

10. $\displaystyle\lim_{x\to 5} 9$

Finding a Limit **In Exercises 11–28, find the limit.**

11. $\displaystyle\lim_{x\to -6} x^2$

12. $\displaystyle\lim_{x\to 0} (5x-3)$

13. $\displaystyle\lim_{t\to 4} \sqrt{t+2}$

14. $\displaystyle\lim_{x\to -5} \sqrt[3]{x-3}$

15. $\displaystyle\lim_{x\to 6} (x-2)^2$

16. $\displaystyle\lim_{x\to 7} (x-4)^3$

17. $\displaystyle\lim_{x\to 4} \frac{4}{x-1}$

18. $\displaystyle\lim_{x\to 2} \frac{x}{x^2+1}$

19. $\displaystyle\lim_{t\to -2} \frac{t+2}{t^2-4}$

20. $\displaystyle\lim_{t\to 4} \frac{t^2-16}{t-4}$

21. $\displaystyle\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4}$

22. $\displaystyle\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

23. $\displaystyle\lim_{x\to 0} \frac{[1/(x+1)]-1}{x}$

24. $\displaystyle\lim_{s\to 0} \frac{\left(1/\sqrt{1+s}\right)-1}{s}$

25. $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{\sin x}$

26. $\displaystyle\lim_{x\to \pi/4} \frac{4x}{\tan x}$

27. $\displaystyle\lim_{\Delta x\to 0} \frac{\sin[(\pi/6)+\Delta x]-(1/2)}{\Delta x}$

[*Hint:* $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$]

28. $\displaystyle\lim_{\Delta x\to 0} \frac{\cos(\pi+\Delta x)+1}{\Delta x}$

[*Hint:* $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$]

Evaluating a Limit **In Exercises 29–32, evaluate the limit given $\lim_{x\to c} f(x) = -6$ and $\lim_{x\to c} g(x) = \frac{1}{2}$.**

29. $\displaystyle\lim_{x\to c} [f(x)g(x)]$

30. $\displaystyle\lim_{x\to c} \frac{f(x)}{g(x)}$

31. $\displaystyle\lim_{x\to c} [f(x)+2g(x)]$

32. $\displaystyle\lim_{x\to c} [f(x)]^2$

Graphical, Numerical, and Analytic Analysis **In Exercises 33–36, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

33. $\displaystyle\lim_{x\to 0} \frac{\sqrt{2x+9}-3}{x}$

34. $\displaystyle\lim_{x\to 0} \frac{[1/(x+4)]-(1/4)}{x}$

35. $\displaystyle\lim_{x\to -5} \frac{x^3+125}{x+5}$

36. $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{x}$

Free-Falling Object **In Exercises 37 and 38, use the position function $s(t) = -4.9t^2 + 250$, which gives the height (in meters) of an object that has fallen for t seconds from a height of 250 meters. The velocity at time $t = a$ seconds is given by**

$$\lim_{t\to a} \frac{s(a)-s(t)}{a-t}.$$

37. Find the velocity of the object when $t = 4$.

38. At what velocity will the object impact the ground?

Finding a Limit **In Exercises 39–48, find the limit (if it exists). If it does not exist, explain why.**

39. $\displaystyle\lim_{x\to 3^+} \frac{1}{x+3}$

40. $\displaystyle\lim_{x\to 6^-} \frac{x-6}{x^2-36}$

41. $\displaystyle\lim_{x\to 4^-} \frac{\sqrt{x}-2}{x-4}$

42. $\displaystyle\lim_{x\to 3^-} \frac{|x-3|}{x-3}$

43. $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} (x-2)^2, & x \le 2 \\ 2-x, & x > 2 \end{cases}$

44. $\lim_{x\to 1^+} g(x)$, where $g(x) = \begin{cases} \sqrt{1-x}, & x \le 1 \\ x+1, & x > 1 \end{cases}$

45. $\lim_{t\to 1} h(t)$, where $h(t) = \begin{cases} t^3+1, & t < 1 \\ \frac{1}{2}(t+1), & t \ge 1 \end{cases}$

46. $\lim_{s\to -2} f(s)$, where $f(s) = \begin{cases} -s^2-4s-2, & s \le -2 \\ s^2+4s+6, & s > -2 \end{cases}$

47. $\lim_{x\to 2^-} (2[\![x]\!]+1)$

48. $\lim_{x\to 4} [\![x-1]\!]$

Removable and Nonremovable Discontinuities **In Exercises 49–54, find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?**

49. $f(x) = x^2 - 4$

50. $f(x) = x^2 - x + 20$

51. $f(x) = \dfrac{4}{x-5}$

52. $f(x) = \dfrac{1}{x^2-9}$

53. $f(x) = \dfrac{x}{x^3-x}$

54. $f(x) = \dfrac{x+3}{x^2-3x-18}$

55. Making a Function Continuous Determine the value of c such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

56. Making a Function Continuous Determine the values of b and c such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} x+1, & 1 < x < 3 \\ x^2+bx+c, & |x-2| \ge 1 \end{cases}$$

Testing for Continuity **In Exercises 57–62, describe the intervals on which the function is continuous.**

57. $f(x) = -3x^2 + 7$

58. $f(x) = \dfrac{4x^2+7x-2}{x+2}$

59. $f(x) = \sqrt{x-4}$

60. $f(x) = [\![x+3]\!]$

61. $f(x) = \begin{cases} \dfrac{3x^2-x-2}{x-1}, & x \ne 1 \\ 0, & x = 1 \end{cases}$

62. $f(x) = \begin{cases} 5-x, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

63. Using the Intermediate Value Theorem Use the Intermediate Value Theorem to show that $f(x) = 2x^3 - 3$ has a zero in the interval $[1, 2]$.

64. Delivery Charges The cost of sending an overnight package from New York to Atlanta is \$12.80 for the first pound and \$2.50 for each additional pound or fraction thereof. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds. Sketch the graph of this function and discuss its continuity.

65. Finding Limits Let

$$f(x) = \frac{x^2-4}{|x-2|}.$$

Find each limit (if it exists).

(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$

66. Finding Limits Let $f(x) = \sqrt{x(x-1)}$.

(a) Find the domain of f.

(b) Find $\lim_{x\to 0^-} f(x)$.

(c) Find $\lim_{x\to 1^+} f(x)$.

Finding Vertical Asymptotes **In Exercises 67–72, find the vertical asymptotes (if any) of the graph of the function.**

67. $f(x) = \dfrac{3}{x}$

68. $f(x) = \dfrac{5}{(x-2)^4}$

69. $f(x) = \dfrac{x^3}{x^2-9}$

70. $h(x) = \dfrac{6x}{36-x^2}$

71. $g(x) = \dfrac{2x+1}{x^2-64}$

72. $f(x) = \csc \pi x$

Finding a One-Sided Limit **In Exercises 73–82, find the one-sided limit (if it exists).**

73. $\lim_{x\to 1^-} \dfrac{x^2+2x+1}{x-1}$

74. $\lim_{x\to (1/2)^+} \dfrac{x}{2x-1}$

75. $\lim_{x\to -1^+} \dfrac{x+1}{x^3+1}$

76. $\lim_{x\to -1^-} \dfrac{x+1}{x^4-1}$

77. $\lim_{x\to 0^+} \left(x - \dfrac{1}{x^3}\right)$

78. $\lim_{x\to 2^-} \dfrac{1}{\sqrt[3]{x^2-4}}$

79. $\lim_{x\to 0^+} \dfrac{\sin 4x}{5x}$

80. $\lim_{x\to 0^+} \dfrac{\sec x}{x}$

81. $\lim_{x\to 0^+} \dfrac{\csc 2x}{x}$

82. $\lim_{x\to 0^-} \dfrac{\cos^2 x}{x}$

83. Environment A utility company burns coal to generate electricity. The cost C in dollars of removing $p\%$ of the air pollutants in the stack emissions is

$$C = \frac{80{,}000p}{100-p}, \quad 0 \le p < 100.$$

(a) Find the cost of removing 15% of the pollutants.

(b) Find the cost of removing 50% of the pollutants.

(c) Find the cost of removing 90% of the pollutants.

(d) Find the limit of C as p approaches 100 from the left and interpret its meaning.

84. Limits and Continuity The function f is defined as shown.

$$f(x) = \frac{\tan 2x}{x}, \quad x \ne 0$$

(a) Find $\lim_{x\to 0} \dfrac{\tan 2x}{x}$ (if it exists).

(b) Can the function f be defined at $x = 0$ such that it is continuous at $x = 0$?